Chapter-1	Double Integrals
Topic-1.3-A	

Change of Order of Integration

The concept of change of order of integration evolved to help in handling typical integrals occurring in evaluation of double integrals. If it is difficult to evaluate double integral with given limits, we change the order of integration.

Working Rule: To Change Order of Integration of the Type $\int_{x=a}^{x=b} \int_{y=h(x)}^{y=k(x)} f(x,y)dxdy$

1. **Using given limits, draw the region of integration.**
   In this case, order is YX. Thus, initially strip is parallel to Y axis.
   Lower limit of inner integral is $y = h(x)$ $\Rightarrow$ lower end of strip is touching the curve $y = h(x)$
   Upper limit of inner integral is $y = k(x)$ $\Rightarrow$ Upper end of strip is touching the curve $y = k(x)$
   Limits of outer integral is from $x = a$ to $x = b$ $\Rightarrow$ Strip moves from $x = a$ to $x = b$.
   Using this information draw the region.

2. **Change the direction of strip in the region drawn**. In this case, now we consider a strip parallel to X axis. Thus, order of integration is XY.

3. **Write limits of integration and Evaluate.**

Working Rule: To Change Order of Integration of the Type $\int_{y=c}^{y=d} \int_{x=u(y)}^{x=v(y)} f(x,y)dxdy$

1. **Using given limits, draw the region of integration.**
   In this case order is XY. Thus, initially strip is parallel to X axis.
   Lower limit of inner integral is $x = u(y)$ $\Rightarrow$ lower end of strip is touching the curve $x = u(y)$
   Upper limit of inner integral is $x = v(y)$ $\Rightarrow$ Upper end of strip is touching the curve $x = v(y)$
   Limits of outer integral is from $y = c$ to $y = d$ $\Rightarrow$ Strip moves from $y = c$ to $y = d$.
   Using this information draw the region.

2. **Change the direction of strip in the region drawn**. In this case, now we consider a strip parallel to Y axis. Thus, order of integration is YX.

3. **Write limits of integration and Evaluate.**

Type 3 Change the order of integration & evaluate

(1.3-A) Solved Examples

1. $\int_{0}^{1}\int_{0}^{\sqrt{1-x^2}} \frac{e^y}{(e^y+1)\sqrt{1-x^2-y^2}}\,dxdy$

2. $\int_{0}^{1}\int_{0}^{\sqrt{1-y^2}} \frac{x}{(1+x^2)\sqrt{1-x^2-y^2}}\,dxdy$

3. $\int_{0}^{\pi/2}\int_{0}^{y} \cos 2y\sqrt{1-a^2\sin^2 x}\,dxdy$

4. $\int_{0}^{2}\int_{\sqrt{2x}}^{2} \frac{y^2}{\sqrt{y^4-4x^2}}\,dxdy$

5. $\int_{0}^{1/2}\int_{0}^{\sqrt{1-4y^2}} \frac{1+x^2}{\sqrt{1-x^2}\sqrt{1-x^2-y^2}}\,dxdy$

6. $\int_{0}^{a}\int_{0}^{a-\sqrt{a^2-y^2}} \frac{xy\log(x+a)}{(x-a)^2}\,dxdy$

7. $\int_{0}^{1}\int_{4y}^{4} e^{x^2}\,dxdy$

8. $\int_{0}^{\infty}\int_{0}^{x} xe^{-x^2/y}\,dxdy$

Example 1.3A.1: Change the order of integration and evaluate

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \frac{e^y}{(e^y+1)\sqrt{1-x^2-y^2}}\,dxdy$$

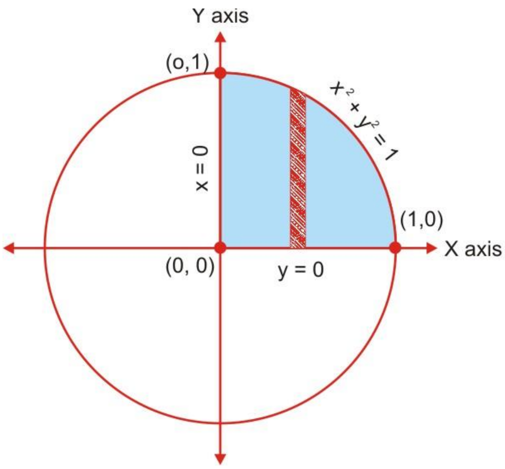
Solution: We draw the region of integration using given limits.

Lower limit of inner integral is $y=0$. This is X axis.

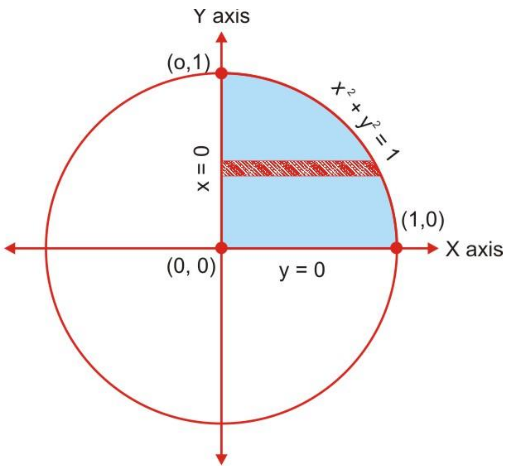
Upper limit of inner integral is $y=\sqrt{1-x^2}$. That is, $x^2+y^2=1$. This is a circle.

Initially, strip is parallel to Y axis, whose lower end touches the X axis and upper end touches the circle $x^2+y^2=1$. This strip moves from left to right end from $x=0$ to $x=1$. Thus, region of integration is first quadrant of the circle $x^2+y^2=1$

Before Changing the order of Integration



After Changing the order of Integration



$$I=\int_{y=0}^{y=1}\int_{x=0}^{x=\sqrt{1-y^2}} \frac{e^y}{(e^y+1)\sqrt{1-x^2-y^2}}\,dxdy$$

$$=\int_{y=0}^{y=1}\frac{e^y}{(e^y+1)}\left[\int_{x=0}^{x=\sqrt{1-y^2}}\frac{1}{\sqrt{(1-y^2)-x^2}}\,dx\right]dy$$

$$=\int_{y=0}^{y=1}\frac{e^y}{(e^y+1)}\left[\sin^{-1}\left(\frac{x}{\sqrt{1-y^2}}\right)\right]_{x=0}^{x=\sqrt{1-y^2}} dy \quad \left\{\because \int\frac{1}{\sqrt{b^2-x^2}}dy=\sin^{-1}\left(\frac{x}{b}\right)\right\}$$

$$=\int_{y=0}^{y=1}\frac{e^y}{(e^y+1)}\left[\sin^{-1}(1)-\sin^{-1}(0)\right]dy$$

$$=\frac{\pi}{2}\int_{y=0}^{y=1}\frac{e^y}{(e^y+1)}dy \quad \left\{\because \sin^{-1}(1)=\frac{\pi}{2}\right\}$$

$$\therefore I = \frac{\pi}{2}\left[\log\left(e^{y}+1\right)\right]_{y=0}^{y=1} \qquad \left\{\text{Using } \int \frac{f'(y)}{f(y)}\,dy = \log f(y)\right\}$$

$$= \frac{\pi}{2}\left[\log\left(e+1\right) - \log\left(1+1\right)\right] = \frac{\pi}{2}\log\left(\frac{e+1}{2}\right)$$

$$\therefore \boxed{\int_0^1 \int_0^{\sqrt{1-x^2}} \frac{e^{y}}{(e^{y}+1)\sqrt{1-x^2-y^2}}\,dxdy = \frac{\pi}{2}\log\left(\frac{e+1}{2}\right)}$$

Example 1.3A.2: Change the order of integration and evaluate

$$\int_0^1 \int_0^{\sqrt{1-y^2}} \frac{x}{(1+x^2)\sqrt{1-x^2-y^2}}dxdy$$

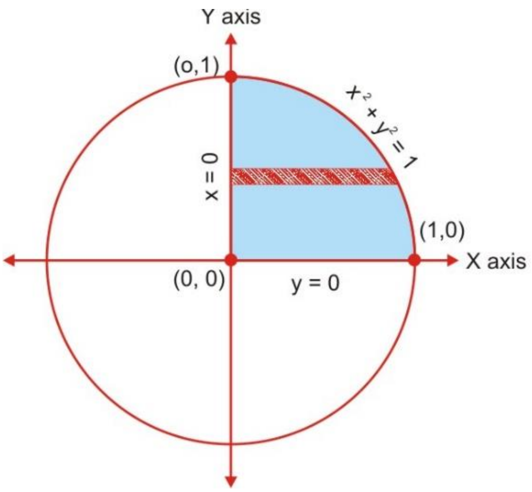
Solution: We draw the region of integration using given limits.

Lower limit of inner integral is $x=0$. This is Y axis.

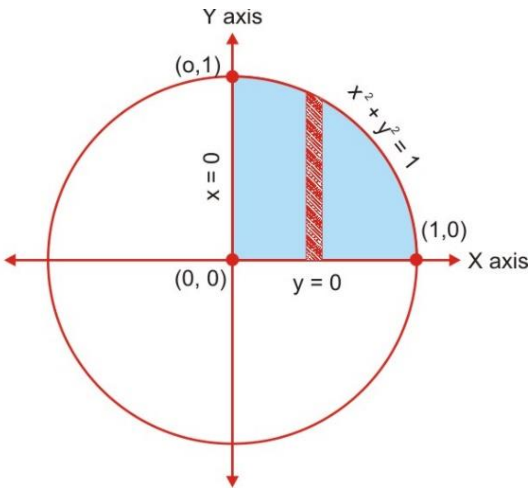
Upper limit of inner integral is $x=\sqrt{1-y^2}$. That is, $x^2+y^2=1$. This is a circle having centre at $(0,0)$ and radius =1.

Initially, strip is parallel to X axis, whose left end touches the Y axis and right end touches the circle $x^2+y^2=1$. This strip moves from lower to upper end from $y=0$ to $y=1$. Thus, region of integration is first quadrant of the circle $x^2+y^2=1$

Before Changing the order of Integration



After Changing the order of Integration



$$I=\int_{x=0}^{x=1}\int_{y=0}^{y=\sqrt{1-x^2}} \frac{x}{\left(1+x^2\right)\sqrt{1-x^2-y^2}}dxdy$$

$$=\int_{x=0}^{x=1}\frac{x}{\left(1+x^2\right)}\left[\int_{y=0}^{y=\sqrt{1-x^2}}\frac{1}{\sqrt{1-x^2-y^2}}dy\right]dx$$

$$=\int_0^1\frac{x}{\left(1+x^2\right)}\left[\sin^{-1}\left(\frac{y}{\sqrt{1-x^2}}\right)\right]_{y=0}^{y=\sqrt{1-x^2}}dx \quad \left\{\because \int\frac{1}{\sqrt{b^2-y^2}}dy=\sin^{-1}\left(\frac{y}{b}\right)\right\}$$

$$=\int_0^1\frac{x}{\left(1+x^2\right)}\left[\sin^{-1}(1)-\sin^{-1}(0)\right]dx$$

$$\therefore I = \int_{x=0}^{x=1} \frac{x}{\left(1+x^2\right)}\left[\frac{\pi}{2}-0\right]dx$$

$$= \frac{\pi}{2}\cdot\frac{1}{2}\int_{x=0}^{x=1} \frac{2x}{\left(1+x^2\right)}dx$$

$$= \frac{\pi}{4}\left[\log\left(1+x^2\right)\right]_{x=0}^{x=1} \quad \left\{\text{Using } \int \frac{f'(x)}{f(x)}dx = \log f(x)\right\}$$

$$= \frac{\pi}{4}\left[\log 2 - \log 1\right] = \frac{\pi}{4}\log 2$$

$$\therefore \boxed{\int_0^1 \int_0^{\sqrt{1-y^2}} \frac{x}{(1+x^2)\sqrt{1-x^2-y^2}}dxdy = \frac{\pi}{4}\log 2}$$

Example 1.3A.3: Change the order of integration and evaluate $\int_0^{\pi/2}\int_0^{y}\cos 2y\sqrt{1-a^2\sin^2 x}\,dx\,dy$

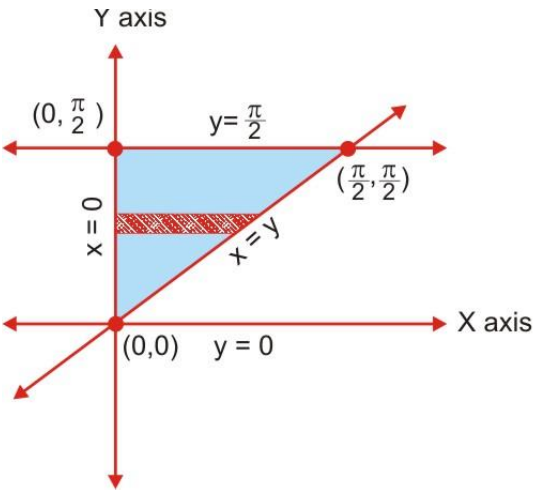
Solution: We draw the region of integration using given limits.

Lower limit of inner integral is $x=0$. This is Y axis.

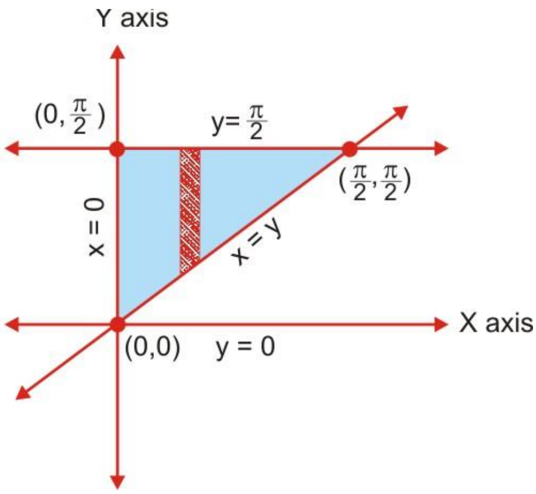
Upper limit of inner integral is $x=y$. This is a straight line passing through $(0,0)$ and $(1,1)$

Initially, strip is parallel to X axis, whose left end touches the Y axis and right end touches the straight line $x=y$. This strip moves from lower to upper end from $y=0$ to $y=\frac{\pi}{2}$.

Before Changing the order of integration



After Changing the order of integration



$$I=\int_{x=0}^{x=\pi/2}\int_{y=x}^{y=\pi/2}\cos 2y\sqrt{1-a^2\sin^2 x}\,dx\,dy$$

$$=\int_{x=0}^{x=\pi/2}\sqrt{1-a^2\sin^2 x}\left[\int_{y=x}^{y=\pi/2}\cos 2y\,dy\right]dx$$

$$=\int_{x=0}^{x=\pi/2}\sqrt{1-a^2\sin^2 x}\left[\frac{\sin 2y}{2}\right]_{y=x}^{y=\pi/2}dx$$

$$=\frac{1}{2}\int_{x=0}^{x=\pi/2}\sqrt{1-a^2\sin^2 x}\left[\sin\pi-\sin 2x\right]dx$$

$$=\frac{1}{2}\int_{x=0}^{x=\pi/2}\sqrt{1-a^2\sin^2 x}\left[-\sin 2x\right]dx$$

Put $1-a^2\sin^2 x = t$

$\therefore -a^2 2\sin x\cos x dx = dt$ or $-\sin 2x dx = \dfrac{1}{a^2}dt$

x	0	$\pi/2$
t	1	$1-a^2$

$$\therefore I = \frac{1}{2a^2}\int_{t=1}^{t=1-a^2}\sqrt{t}\,\frac{dt}{a^2}$$

$$= \frac{1}{2a^2}\left[\frac{t^{3/2}}{3/2}\right]_{t=1}^{t=1-a^2}$$

$$= \frac{1}{3a^2}\left[\left(1-a^2\right)^{3/2}-1\right]$$

$$\therefore \boxed{\int_0^{\pi/2}\int_0^{y}\cos 2y\sqrt{1-a^2\sin^2 x}\,dx\,dy = \frac{1}{3a^2}\left[\left(1-a^2\right)^{3/2}-1\right]}$$

Example 1.3A.4: Change the order of integration and evaluate $\int_0^2 \int_{\sqrt{2x}}^{2} \frac{y^2}{\sqrt{y^4-4x^2}} dxdy$

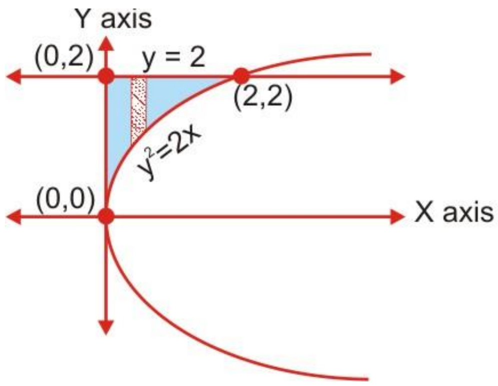
Solution: We draw the region of integration using given limits.

Lower limit of inner integral is $y=\sqrt{2x}$. That is, $y^2=2x$. This is a parabola symmetrical about X axis.

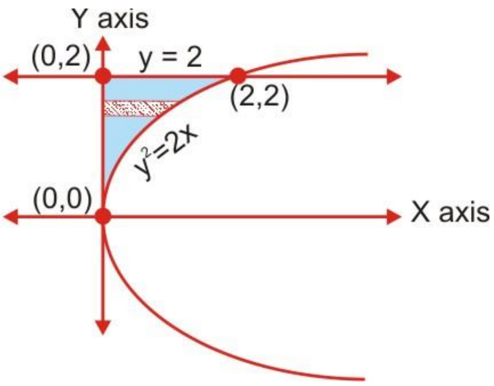
Upper limit of inner integral is $y=2$. This is a straight line parallel to X axis and passing through $(0,2)$.

Initially, strip is parallel to Y axis, whose lower end touches the parabola $y^2=2x$ and upper end touches the straight line $y=2$. This strip moves from left to right from $x=0$ to $x=2$.

Before Changing the order of Integration



After Changing the order of Integration



$$I=\int_0^2 y^2\left[\int_{x=0}^{x=y^2/2} \frac{1}{\sqrt{(y^2)^2-(2x)^2}}dx\right]dy$$

$$=\int_0^2 y^2\left[\frac{1}{2}\sin^{-1}\left(\frac{2x}{y^2}\right)\right]_{x=0}^{x=y^2/2} dy \qquad \left\{\because \int\frac{1}{\sqrt{a^2-x^2}}dx=\sin^{-1}\left(\frac{x}{a}\right)\right\}$$

$$=\frac{1}{2}\int_0^2 y^2\left[\sin^{-1}(1)-\sin^{-1}(0)\right]dy$$

$$=\frac{1}{2}\int_0^2 y^2\left[\frac{\pi}{2}\right]dy=\frac{\pi}{4}\left[\frac{y^3}{3}\right]_0^2=\frac{8\pi}{12}=\frac{2\pi}{3}$$

$$\therefore \boxed{\int_0^2\int_{\sqrt{2x}}^{2}\frac{y^2}{\sqrt{y^4-4x^2}}dxdy=\frac{2\pi}{3}}$$

Example 1.3A.5: Change the order of integration and evaluate

$$\int_0^{1/2}\int_0^{\sqrt{1-4y^2}} \frac{1+x^2}{\sqrt{1-x^2}\sqrt{1-x^2-y^2}}dxdy$$

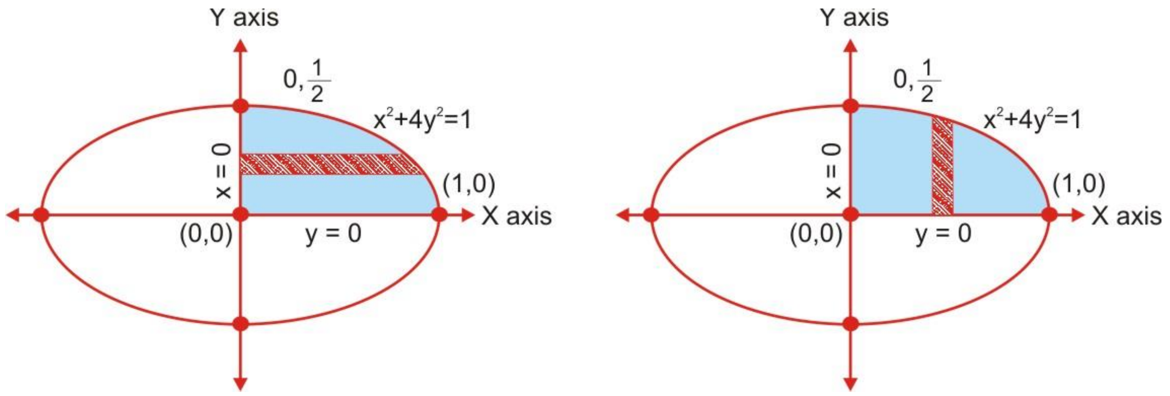
Solution: We draw the region of integration using given limits.

Lower limit of inner integral is $x=0$. This is Y axis.

Upper limit of inner integral is $x=\sqrt{1-4y^2}$. That is, $x^2+4y^2=1$. This is an ellipse.

Initially, strip is parallel to X axis, whose left end touches the Y axis and right end touches the ellipse $x^2+4y^2=1$. This strip moves from lower to upper end from $y=0$ to $y=\frac{1}{2}$.

Before Changing the order of Integration **After Changing the order of Integration**



After changing the order of integration

$$I=\int_{x=0}^{x=1}\int_{y=0}^{y=\left(\frac{\sqrt{1-x^2}}{2}\right)} \frac{1+x^2}{\sqrt{1-x^2}\sqrt{1-x^2-y^2}}dxdy$$

$$=\int_0^1 \frac{1+x^2}{\sqrt{1-x^2}}\left[\int_{y=0}^{y=\left(\frac{\sqrt{1-x^2}}{2}\right)} \frac{1}{\sqrt{1-x^2-y^2}}dy\right]dx$$

$$=\int_0^1 \frac{1+x^2}{\sqrt{1-x^2}}\left[\sin^{-1}\left(\frac{y}{\sqrt{1-x^2}}\right)\right]_{y=0}^{y=\left(\frac{\sqrt{1-x^2}}{2}\right)} dx \quad \left\{\because \int \frac{1}{\sqrt{b^2-y^2}}dy=\sin^{-1}\left(\frac{y}{b}\right)\right\}$$

$$\therefore I=\int_0^1 \frac{1+x^2}{\sqrt{1-x^2}}\left[\sin^{-1}\left(\frac{1}{2}\right)-\sin^{-1}(0)\right]dx$$

$$=\frac{\pi}{6}\int_0^1 \frac{1+x^2}{\sqrt{1-x^2}}dx \quad \left\{\because \sin^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{6}\right\}$$

Put $x^2=t,\ \therefore x=t^{1/2}$ and $dx=\frac{1}{2}t^{-1/2}dt$

x	0	1
t	0	1

$$I=\frac{\pi}{6}\int_0^1 \frac{1+t}{\sqrt{1-t}}\frac{1}{2}t^{-1/2}dt$$

$$I=\frac{\pi}{12}\left[\int_0^1 t^{-1/2}(1-t)^{-1/2}dt+\int_0^1 t^{1/2}(1-t)^{-1/2}dt\right]$$

$$=\frac{\pi}{12}\left[\beta\left(\frac{1}{2},\frac{1}{2}\right)+\beta\left(\frac{3}{2},\frac{1}{2}\right)\right] \quad \left\{\because \int_0^1 t^m(1-t)^n dt=\beta(m+1,n+1)\right\}$$

$$=\frac{\pi}{12}\left[\frac{\Gamma\left(\frac{1}{2}\right)\Gamma\left(\frac{1}{2}\right)}{\Gamma(1)}+\frac{\Gamma\left(\frac{3}{2}\right)\Gamma\left(\frac{1}{2}\right)}{\Gamma(2)}\right]$$

$$=\frac{\pi}{12}\left[\frac{\sqrt{\pi}\sqrt{\pi}}{1}+\frac{\frac{1}{2}\sqrt{\pi}\sqrt{\pi}}{1}\right]$$

$$=\frac{\pi}{12}\left[\pi+\frac{\pi}{2}\right]=\frac{\pi}{12}\left[\frac{3\pi}{2}\right]=\frac{\pi^2}{8}$$

$$\therefore \boxed{\int_0^{1/2}\int_0^{\sqrt{1-4y^2}} \frac{1+x^2}{\sqrt{1-x^2}\sqrt{1-x^2-y^2}}dxdy=\frac{\pi^2}{8}}$$

Example 1.3A.6: Change the order of integration and evaluate $\int_0^a \int_0^{a-\sqrt{a^2-y^2}} \frac{xy\log(x+a)}{(x-a)^2}dxdy$

Solution: We draw the region of integration using given limits.

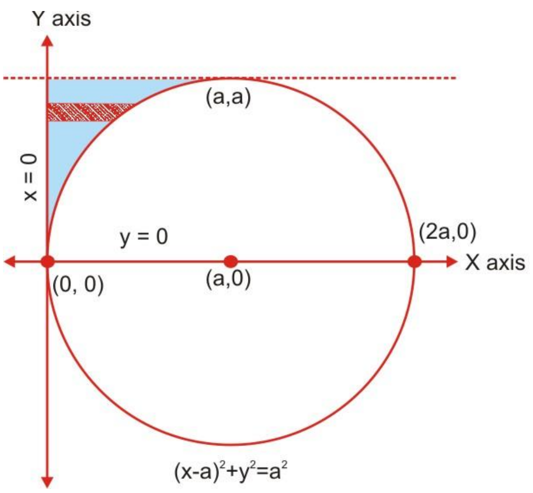
Lower limit of inner integral is $x=0$. This is Y axis.

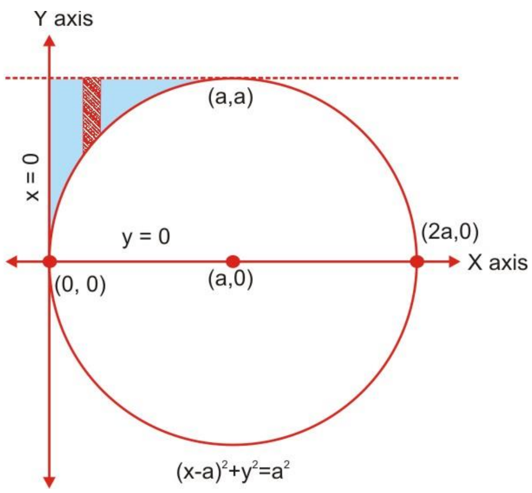
Upper limit of inner integral is $x=a-\sqrt{a^2-y^2}$. That is, $(x-a)^2+y^2=a^2$. This is a circle having centre at $(a,0)$ and radius = a.

Initially, strip is parallel to X axis, whose left end touches the Y axis and right end touches the circle $(x-a)^2+y^2=a^2$. This strip moves from lower to upper end from $y=0$ to $y=a$.

Note that $x=a-\sqrt{a^2-y^2}$ implies that the right end of the strip will touch that part of the circle for which $x \le a$

Before Changing the order of Integration **After Changing the order of Integration**





After changing the order of integration

$$I = \int_{x=0}^{x=a} \int_{y=\sqrt{2ax-x^2}}^{y=a} \frac{xy\log(x+a)}{(x-a)^2}dxdy$$

$$= \int_{x=0}^{x=a} \frac{x\log(x+a)}{(x-a)^2}\left[\int_{y=\sqrt{2ax-x^2}}^{y=a} ydy\right]dx$$

$$= \int_{x=0}^{x=a} \frac{x\log(x+a)}{(x-a)^2}\left[\frac{y^2}{2}\right]_{y=\sqrt{2ax-x^2}}^{y=a} dx$$

$$\therefore I=\frac{1}{2}\int_{x=0}^{x=a}\frac{x\log(x+a)}{(x-a)^2}\left[a^2-\left(2ax-x^2\right)\right]dx$$

$$=\frac{1}{2}\int_{x=0}^{x=a}\frac{x\log(x+a)}{(x-a)^2}(x-a)^2\,dx$$

$$=\frac{1}{2}\int_{x=0}^{x=a}x\log(x+a)dx$$

$$=\frac{1}{2}\int_{x=0}^{x=a}\log(x+a)\cdot xdx$$

$$=\frac{1}{2}\left\{\log(x+a)\int xdx-\int\left[\frac{d}{dx}\log(x+a)\cdot\int xdx\right]dx\right\}_{x=0}^{x=a}\quad\text{Integrating by parts}$$

$$=\frac{1}{2}\left\{\log(x+a)\cdot\frac{x^2}{2}-\int\left[\frac{1}{x+a}\cdot\frac{x^2}{2}\right]dx\right\}_{x=0}^{x=a}$$

$$=\frac{1}{4}\left\{x^2\cdot\log(x+a)-\int\frac{\left(x^2-a^2\right)+a^2}{x+a}dx\right\}_{x=0}^{x=a}$$

$$=\frac{1}{4}\left\{x^2\cdot\log(x+a)-\int\frac{(x-a)(x+a)+a^2}{x+a}dx\right\}_{x=0}^{x=a}$$

$$=\frac{1}{4}\left\{x^2\cdot\log(x+a)-\int\left[(x-a)+\frac{a^2}{x+a}\right]dx\right\}_{x=0}^{x=a}$$

$$=\frac{1}{4}\left\{x^2\cdot\log(x+a)-\frac{(x-a)^2}{2}-a^2\log(x+a)\right\}_{x=0}^{x=a}$$

$$=\frac{1}{4}\left\{\left[a^2\cdot\log(2a)-0-a^2\log(2a)\right]-\left[0-\frac{a^2}{2}-a^2\log a\right]\right\}$$

$$=\frac{1}{4}\left[\frac{a^2}{2}+a^2\log a\right]=\frac{a^2}{8}\left(1+2\log a\right)$$

$$\therefore \boxed{\int_0^a\int_0^{a-\sqrt{a^2-y^2}}\frac{xy\log(x+a)}{(x-a)^2}dxdy=\frac{a^2}{8}\left(1+2\log a\right)}$$

Example 1.3A.7: Change the order of integration and evaluate $\int_0^1 \int_{4y}^{4} e^{x^2} dxdy$

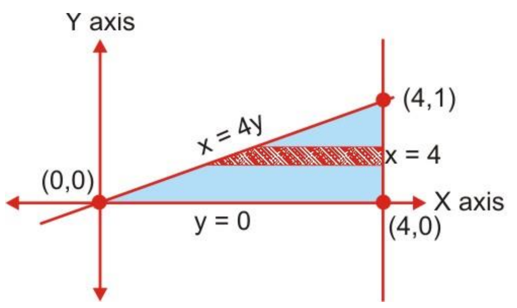
Solution: We draw the region of integration using given limits.

Lower limit of inner integral is $x = 4y$. This is a straight line passing through $(0,0)$ and $(4,1)$

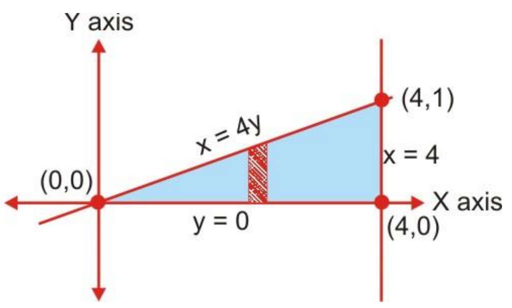
Upper limit of inner integral is $x = 4$. This is a straight line parallel to Y axis and passing through $(4,0)$

Initially, strip is parallel to X axis, whose left end touches the straight line $x = 4y$ and right end touches the straight line $x = 4$. This strip moves from lower to upper end from $y = 0$ to $y = 1$.

Before Changing the order of Integration



After Changing the order of Integration



$$I = \int_{x=0}^{x=4} \int_{y=0}^{y=\frac{x}{4}} e^{x^2} dxdy = \int_{x=0}^{x=4} e^{x^2} \left[\int_{y=0}^{y=\frac{x}{4}} dy \right] dx$$

$$= \int_{x=0}^{x=4} e^{x^2} [y]_0^{x/4} dx = \int_{x=0}^{x=4} e^{x^2} \left[\frac{x}{4}\right] dx$$

$$= \frac{1}{4} \int_{x=0}^{x=4} e^{x^2} xdx$$

Put $x^2 = t$, $\therefore 2xdx = dt$

x	0	4
t	0	16

$$\therefore I = \frac{1}{4} \int_0^{16} e^t \left(\frac{dt}{2}\right) = \frac{1}{8}\left[e^t\right]_0^{16} = \frac{1}{8}\left[e^{16} - 1\right]$$

$$\therefore \boxed{\int_0^1 \int_{4y}^{4} e^{x^2} dxdy = \frac{1}{8}\left[e^{16} - 1\right]}$$

Example 1.3A.8: Change the order of integration and evaluate $\int_0^{\infty}\int_0^{x} xe^{-x^2/y}\,dxdy$

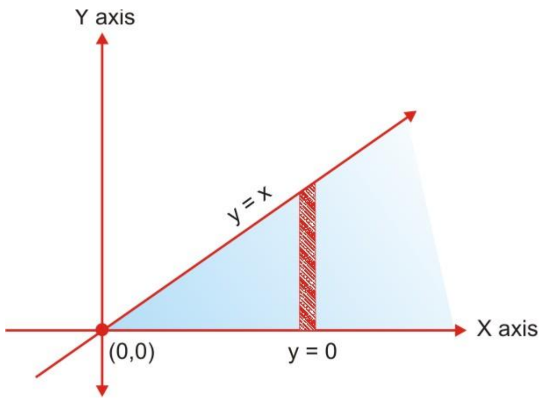
Solution: We draw the region of integration using given limits. $I=\int_{x=0}^{x=\infty}\int_{y=0}^{y=x} xe^{-x^2/y}\,dxdy$

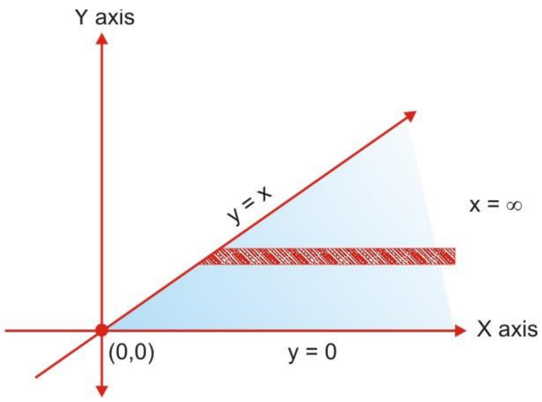
Lower limit of inner integral is $y=0$. This is X axis.

Upper limit of inner integral is $y=x$. This is a straight line passing through $(0,0)$

Initially, strip is parallel to Y axis, whose lower end touches the X axis and upper end touches the straight line $y=x$. This strip moves from left to right end from $x=0$ to $x=\infty$.

Before Changing the order of Integration **After Changing the order of Integration**





$$I=\int_{y=0}^{y=\infty}\int_{x=y}^{x=\infty} xe^{-x^2/y}\,dxdy=\int_{y=0}^{y=\infty}\left[\int_{x=y}^{x=\infty} e^{-x^2/y}\,xdx\right]dy----(1)$$

Consider $I_1=\int_{x=y}^{x=\infty} e^{-x^2/y}\,xdx$

Put $\frac{x^2}{y}=t \quad \therefore \frac{2x}{y}dx=dt$ or $xdx=\frac{y}{2}dt$

x	y	∞
t	y	∞

$$\therefore I_1=\int_{t=y}^{t=\infty} e^{-t}\frac{y}{2}dt=\frac{y}{2}\left[-e^{-t}\right]_{t=y}^{t=\infty}$$

$$=-\frac{y}{2}\left[e^{-\infty}-e^{-y}\right]$$

$$=-\frac{y}{2}\left[0-e^{-y}\right]=\frac{1}{2}ye^{-y}----(2)$$

From (1) and (2),

$$I = \frac{1}{2}\int_{y=0}^{y=\infty} ye^{-y}dy$$

$$= \frac{1}{2}\left[y\left(-e^{-y}\right) - 1\cdot\left(e^{-y}\right)\right]_{y=0}^{y=\infty}$$

$$= \frac{1}{2}\left[0-0-0+1\right] = \frac{1}{2}$$

$$\therefore \boxed{\int_0^\infty \int_0^x xe^{-x^2/y}dxdy = \frac{1}{2}}$$